Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. A particle A of mass $3m$ and a particle B of mass m are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of A is ku and the speed of B is u. Immediately after the collision, the speed of A is v and the speed of B is $2v$.

The magnitude of the impulse received by B in the collision is $\frac{3}{2}mu$.

(a) Find v in terms of u only. **(3)**

(b) Find the two possible values of k. **(5)**

(a)

Impulse is the change in momentum

Formula for change in momentum:

mass

$I = \Delta\text{momentum} = mv_{\text{final}} - mv_{\text{initial}}$

velocity

Substitute impulse on B:

pay attention to sign/direction

M1 $I = m(2v - (-u))$

magnitude of I, |I|

$\frac{3}{2}\cancel{m}u = \cancel{m}(2v + u)$ A1

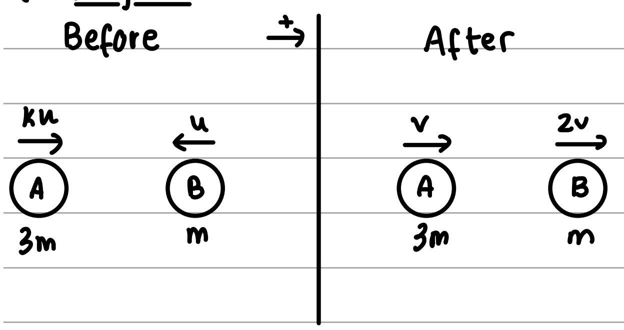
$\frac{3}{2}u = 2v + u$

$\frac{1}{2}u = 2v$

$v = \frac{u}{4}$ A1

Question 1 continued

(b) Diagram



We can use the conservation of linear momentum to get this.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A \underbrace{u_A}_{\text{initial velocity}} + m_B u_B = m_A \underbrace{v_A}_{\text{final velocity}} + m_B v_B$

Substitute:

$$3\cancel{m}(ku) + \cancel{m}(-u) = 3\cancel{m}(v) + \cancel{m}(2v)$$ cancel m's M1A1

$$3ku - u = 3v + 2v$$

$$3ku - u = 5v$$

Substitute in $v = \frac{u}{4}$:

$$3ku - u = 5 \times \frac{u}{4}$$

$$\cancel{u}(3k-1) = \frac{5\cancel{u}}{4}$$ factor out u, cancel u's

$$3k = \frac{5}{4} + 1$$

$$3k = \frac{9}{4}$$

$$k = \frac{3}{4}$$ value for k A1

to get another value for k, consider the option that A changes direction:

use CLM again:

$$3\cancel{m}(ku) + \cancel{m}(-u) = 3\cancel{m}(-v) + \cancel{m}(2v)$$ M1

$$3ku - u = 2v - 3v$$

$$3ku - u = -v$$

Substitute in $v = \frac{u}{4}$:

$$\cancel{u}(3k-1) = -\frac{\cancel{u}}{4}$$

$$3k - 1 = -\frac{1}{4}$$

$$3k = \frac{3}{4}$$

$$k = \frac{1}{4}$$ value of k A1

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2.

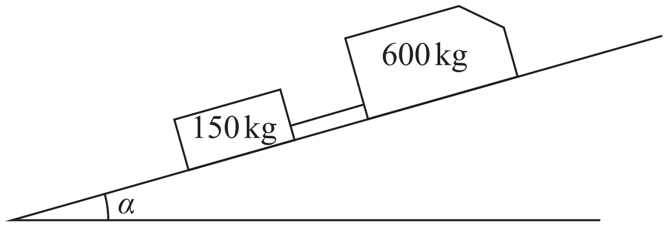


Figure 1

A van of mass 600 kg is moving up a straight road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{15}$. The van is towing a trailer of mass 150 kg. The van is attached to the trailer by a towbar which is parallel to the direction of motion of the van and the trailer, as shown in Figure 1.

The resistance to the motion of the van from non-gravitational forces is modelled as a constant force of magnitude 200 N.
The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 100 N.

The towbar is modelled as a light rod.

The engine of the van is working at a constant rate of 12 kW.

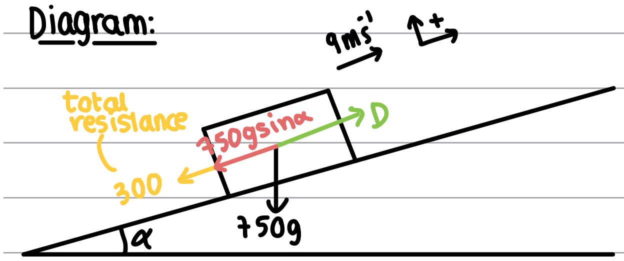
Find the tension in the towbar at the instant when the speed of the van is $9\,\mathrm{m\,s^{-1}}$

(8)

Question 2 continued

First consider the motion as a system:

Diagram:



Use $\Sigma F_x = ma$: ("at the instant when v=9" ∴ we know it accelerates)

$D - 300 - 750g\sin\alpha = 750a$ M1A1

$\frac{4000}{3} - 300 - \frac{750}{15}g = 750a$ we will get a

$\frac{1}{750}\left(\frac{4000}{3} - 300 - 50g\right) = a$ A1

To get D we will use Power.

Formula for Power:

Power (W) — $P = Dv$ — Driving force (N), velocity (ms^{-1})

$P = 12\,kW \xrightarrow{\times 1000} 12000W$

$D = D$

$v = 9\,ms^{-1}$

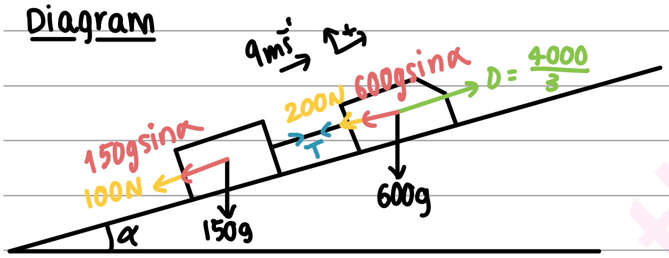
Substitute:

$12\,000 = D(9)$

$D = \frac{4000}{3}$ M1 value for D

Now let's treat the two separately:

Diagram



Use $\Sigma F_x = ma$ on the trailer: M1A1

we want this! $T - 100 - 150g\sin\alpha = 150a$ found above

$T = 150\left(\frac{1}{750}\left(\frac{4000}{3} - 300 - 50g\right)\right) + 100 + \frac{150}{15}g$

Solve for T!

$T = \frac{1}{5}\left(\frac{4000}{3} - 300 - 50g\right) + 100 + 10g$ M1

$= \frac{800}{3} - 60 - \cancel{10g} + 100 + \cancel{10g}$

$= \frac{800}{3} + 40$

$\boxed{T = 307\,N \text{ to } 3sf}$ A1

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3.

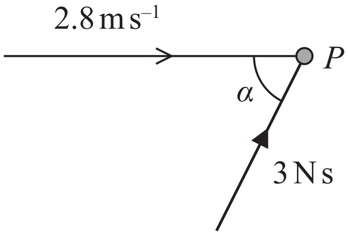


Figure 2

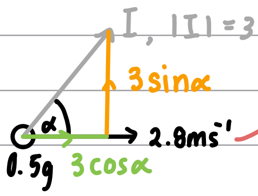
A particle P of mass 0.5 kg is moving in a straight line with speed 2.8 m s^{-1} when it receives an impulse of magnitude 3 N s.
The angle between the direction of motion of P immediately before receiving the impulse and the line of action of the impulse is α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

Find the speed of P immediately after receiving the impulse.

(5)

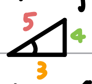
Method 1 – use vectors

Diagram



★ to make our life easier, we will assume the initial velocity is horizontal and we will calculate everything else relatively to that. since we are not given or asked for direction, this is not a problem!

★ If $\tan\alpha = \frac{4}{3}$, using a triangle:



$\sqrt{4^2+3^2} = 5$

$\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$

From the diagram we know that:

1. the speed before as a vector is $\begin{pmatrix} 2.8 \\ 0 \end{pmatrix}$
2. the impulse as a vector is $\begin{pmatrix} 3\cos\alpha \\ 3\sin\alpha \end{pmatrix}$

We can apply the impulse-momentum principle, $I = m(v-u)$, with vectors:

$\begin{pmatrix} 3\cos\alpha \\ 4\sin\alpha \end{pmatrix} = \frac{1}{2}\left(\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} 2.8 \\ 0 \end{pmatrix}\right)$ M1A1

let this be velocity after

Solve separately for a and b:

$3 \times \frac{3}{5} = \frac{1}{2}a - 2.8 \rightarrow a = \frac{32}{5}$

$4 \times \frac{4}{5} = \frac{1}{2}b \rightarrow b = \frac{24}{5}$

$\therefore v = \begin{pmatrix} \frac{32}{5} \\ \frac{24}{5} \end{pmatrix}$ A1

velocity after

Use Pythagoras' Theorem to get the speed, $|v|$.

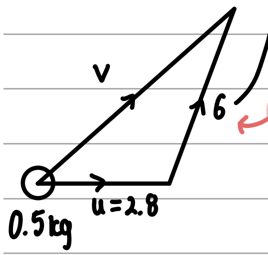
$|v| = \sqrt{\left(\frac{32}{5}\right)^2 + \left(\frac{24}{5}\right)^2}$ M1

$= 8\text{ms}^{-1}$ speed after A1

Question 3 continued

Method 2 - use cosine rule

Diagram

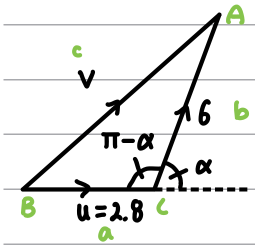


$I = m(v-u)$

$3 = \frac{1}{2}(v-u)$

$6 = v - u \longrightarrow$ From here we can also get that $v = 6 + u$. If you think of these as vectors, you see why the triangle can be constructed

*to make our life easier, we will assume the initial velocity is horizontal and we will calculate everything else relatively to that. Since we are not given or asked for direction, this is not a problem!



Formula for cosine rule M1

$c^2 = a^2 + b^2 - 2ab\cos C$ A1A1

Substitute:

$v^2 = 6^2 + 2.8^2 - 2(6)(2.8)\cos(\pi - \alpha)$

↓ solve for v M1

$v^2 = 36 + 7.84 - 33.6\cos(\pi - \alpha)$ addition formula: $\cos(\alpha - \beta)$

$v^2 = 36 + 7.84 - 33.6\left[\cancel{\cos\pi}^{-1}\cos\alpha + \cancel{\sin\pi}^{0}\sin\alpha\right]$ $= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$v^2 = 36 + 7.84 - 33.6\left(-\frac{3}{5}\right)$

$v^2 = 36 + 7.84 + 20.16$

$v^2 = 64$

$v = 8\text{ms}^{-1}$ speed after A1

(Total for Question 3 is 5 marks)

4.

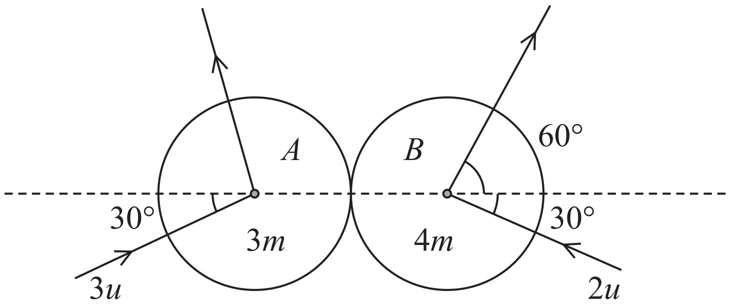


Figure 3

Two smooth uniform spheres, A and B, have equal radii. The mass of A is $3m$ and the mass of B is $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before they collide, A is moving with speed $3u$ at 30° to the line of centres of the spheres and B is moving with speed $2u$ at 30° to the line of centres of the spheres. The direction of motion of B is turned through an angle of 90° by the collision, as shown in Figure 3.

(i) Find the size of the angle through which the direction of motion of A is turned as a result of the collision.

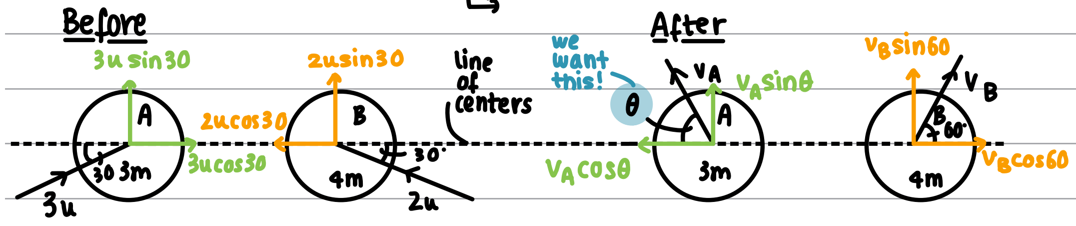
angle of deflection

(ii) Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(9)

Question 4 continued

(i) Diagram



Perpendicular to the line of centers velocities don't change:

$\therefore\ 3u\sin30 = V_A\sin\theta$ and $2u\sin30 = V_B\sin60$ B1 B1

Parallel to the line of centers we will use CLM M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (u_A: initial velocity, v_A: final velocity)

Substitute:

$$3\cancel{m}(3u\cos30) + 4\cancel{m}(-2u\cos30) = 3\cancel{m}(-V_A\cos\theta) + 4\cancel{m}(V_B\cos60) \quad \text{cancel m's}$$

$$9u\left(\frac{\sqrt3}{2}\right) - 8u\left(\frac{\sqrt3}{2}\right) = -3V_A\cos\theta + 4V_B\left(\frac12\right) \quad \text{A1}$$

We can get V_A and V_B from the perpendicular components

$$V_A = \frac{3u\sin30}{\sin\theta} \quad \text{and} \quad V_B = \frac{2u\sin30}{\sin60} = \frac{u}{\frac{\sqrt3}{2}} = \frac{2u}{\sqrt3}$$

Substitute these in:

$$\frac{9\sqrt3\,\cancel{u}}{2} - \frac{8\sqrt3\,\cancel{u}}{2} = -3\left(3\cancel{u}\right)\left(\frac{\sin30}{\sin\theta}\right)\cos\theta + 2\left(\frac{2\cancel{u}}{\sqrt3}\right)$$

$$\frac{\sqrt3}{2} = -9\left(\frac12\right)\cot\theta + \frac{4}{\sqrt3}$$

$$\frac{\sqrt3}{2} - \frac{4\sqrt3}{3} = -\frac92\cot\theta \quad \cot\theta = \frac{1}{\tan\theta} \text{ trigonometry.}$$

$$-\frac{5\sqrt3}{6} = -\frac92\left(\frac{1}{\tan\theta}\right)$$

Rationalize:

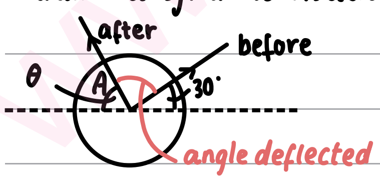
$$\tan\theta = \frac{54}{10\sqrt3} \longrightarrow \frac{\cancel{54}^{27}}{\cancel{10}_{5}\sqrt3} \times \frac{\sqrt3}{\sqrt3} = \frac{\cancel{27}^{9}\sqrt3}{5(\cancel{3})} = \frac{9\sqrt3}{5}$$

$$\tan\theta = \frac{9\sqrt3}{5}$$

$$\theta = \tan^{-1}\left(\frac{9\sqrt3}{5}\right)$$

M1 $\theta = 72.2°$

draw a diagram to visualize the deflection:



$\therefore$ angle deflected $= 180 - \theta - 30$

$= 77.8°$ to 3sf. A1

Question 4 continued

(ii) Impulse is the change in momentum

Formula for change in momentum:

$I = \Delta \text{momentum} = m\underset{\text{final}}{v} - m\underset{\text{initial}}{v}$

mass

velocity

$v_B = \frac{2u}{\sqrt{3}}$ from (i)

Substitute: $|I| = 4m(v\cos 60 - (-2u\cos 30))$ (for B) M1A1

"magnitude" $= 4m\left(\frac{1}{2}\left(\frac{2u}{\sqrt{3}}\right) - \left(-2u\frac{\sqrt{3}}{2}\right)\right)$

$= \frac{16\sqrt{3}}{3}mu$ A1

Question 4 continued

(Total for Question 4 is 9 marks)

5. Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of P is $3m$ and the mass of Q is $4m$.
Immediately before the collision the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{7}(9e+2)$ **(6)**

VQ

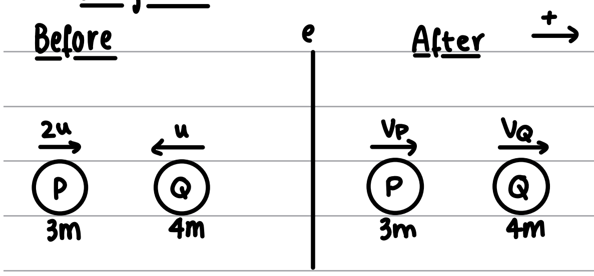
After the collision with P, particle Q collides directly with a fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of Q.

The coefficient of restitution between Q and the wall is $\frac{1}{2}$

(b) Find the complete range of possible values of e for which there is a second collision between P and Q. **(4)**

Question 5 continued

(a) Diagram



We can use the conservation of linear momentum to get an equation. M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

initial velocity   final velocity

Substitute:

$$3\cancel{m}(2u) + 4\cancel{m}(-u) = 3\cancel{m}v_P + 4\cancel{m}v_Q$$ cancel m's

$$6u - 4u = 3v_P + 4v_Q$$

$$2u = 3v_P + 4v_Q$$ Eq1 A1

We can use Newton's Law of Restitution to get another equation. M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution   initial speed   final speed

Substitute:

$$e(2u - (-u)) = v_Q - v_P$$

$$3eu = v_Q - v_P$$ Eq2. A1

Solve simultaneously Eq1 and Eq2: M1

$$2u = 3v_P + 4v_Q$$ add them

$3eu = v_Q - v_P$ |x3| $9eu = -3v_P + 3v_Q$ +   use elimination method.

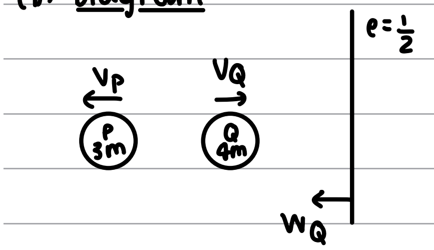
$$9eu + 2u = 7v_Q$$

$$u(9e+2) = 7v_Q$$ factor out u

$$\frac{u}{7}(9e+2) = v_Q$$ hence shown A1

Question 5 continued

(b) Diagram



let's get v_P:

$2u = 3v_P + 4\left(\frac{u}{7}(9e+2)\right)$ v_Q from (a)

$2u = 3v_P + \frac{4u}{7}(9e+2)$

$2u - \frac{36eu}{7} - \frac{8u}{7} = 3v_P$

$\frac{6u}{7} - \frac{36eu}{7} = 3v_P$

$\frac{\cancel{2}^{1}}{7}u(2-12e) = \cancel{3}v_P$

$\frac{u}{7}(2-12e) = v_P$ B1

let's get w_Q by multiplying v_Q by the coefficient of restitution of the wall and by changing its direction.

$w_Q = -\frac{1}{2} \times \frac{u}{7}(9e+2)$

$w_Q = -\frac{u}{14}(9e+2)$ B1

Now we compare these two.

We are moving in the negative direction so we want w_Q to be more negative than v_P, $\therefore$ $w_Q < v_P$ (so that its magnitude is larger than that of v_P) M1

$w_Q < v_P$

$-\frac{\cancel{u}}{14}(9e+2) < \frac{\cancel{u}}{7}(2-12e)$

$9e+2 > -2(2-12e)$

$9e+2 > -4+24e$ careful to switch the inequality sign if you multiply by a negative!

$6 > 15e$

$\div 3$ $\frac{6}{15} > e$ $\div 3$

$\frac{2}{5} > e$

as e is a coefficient of restitution it needs to be smaller than 1.

$\therefore$ $\frac{2}{5} < e \leqslant 1$ complete range of e

A1

Question 5 continued

(Total for Question 5 is 10 marks)

6.

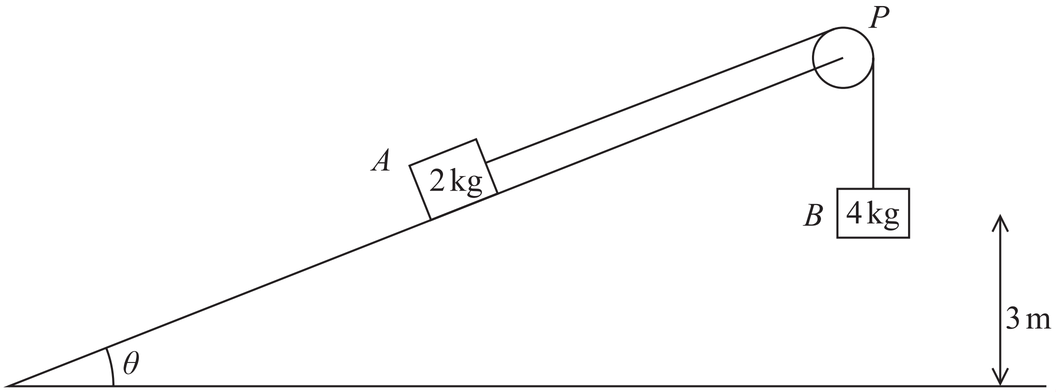


Figure 4

Two blocks, A and B, of masses 2 kg and 4 kg respectively are attached to the ends of a light inextensible string.

Initially A is held on a fixed rough plane. The plane is inclined to horizontal ground at an angle θ, where $\tan\theta = \frac{3}{4}$

The string passes over a small smooth light pulley P that is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane.

Block A is held on the plane with the distance AP greater than 3 m. Block B hangs freely below P at a distance of 3 m above the ground, as shown in Figure 4.

The coefficient of friction between A and the plane is μ

Block A is released from rest with the string taut.

By modelling the blocks as particles,

(a) find the potential energy lost by the whole system as a result of B falling 3 m.

(3)

Given that the speed of B at the instant it hits the ground is 4.5 m s^{-1} and ignoring air resistance,

(b) use the work-energy principle to find the value of μ

(6)

After B hits the ground, A continues to move up the plane but does not reach the pulley in the subsequent motion.
Block A comes to instantaneous rest after moving a total distance of $(3 + d)$ m from its point of release.

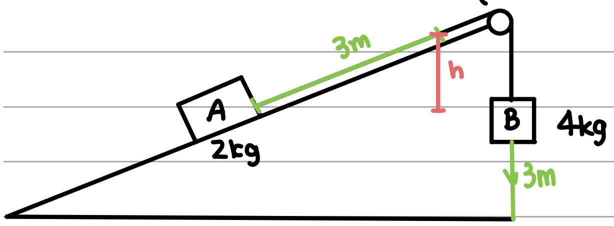
Ignoring air resistance,

(c) use the work-energy principle to find the value of d

(4)

Question 6 continued

(a) the total GPE lost is: (GPE lost by B) − (GPE gained by A) M1



Formula for GPE

GPE = mgh

mass; vertical height change; g = 9.8 ms^{-2}

let's get h :

$\sin\theta = \frac{3}{5} \rightarrow \sin\theta = \frac{h}{3}$

$\frac{3}{5} = \frac{h}{3}$

$\frac{9}{5} = h$ vert. height

Substitute in:

B: $4g(3) - 2g(h)$ A A1

$= 12g - 2g \times \frac{9}{5}$

$= \boxed{82.3\text{J to 3sf}}$ A1

Turn over

Question 6 continued

(b)

★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction).

★ Remember the work-energy formulae:

Either: $\text{WD by force} + KE_i + GPE_i + EPE_i = KE_f + GPE_f + EPE_f + \text{WD against friction}$

work done, initial kinetic, initial grav. potential, initial elastic potential, final kinetic, final grav. potential, final elastic potential, work lost to friction

OR: $\text{WD by force} + KE_i + GPE_i + EPE_i - \text{WD by friction} = KE_f + GPE_f + EPE_f$

work done, initial kinetic, initial grav. potential, initial elastic potential, we subtract this since it leaves the system as heat!, final kinetic, final grav. potential, final elastic potential

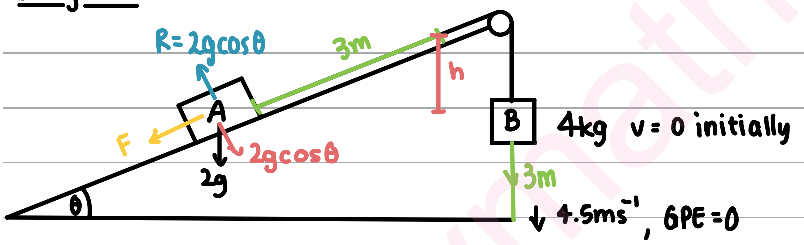
★ Formulae for KE, GPE and EPE:

$KE = \frac{1}{2}mv^2$ — mass, velocity

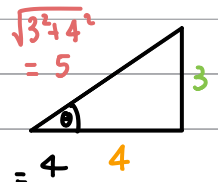
$GPE = mgh$ — mass, change in height, $g = 9.8\,\text{ms}^{-2}$

$EPE = \frac{\lambda x^2}{2l}$ — modulus of elasticity, extension of string/spring, natural length of the string/spring

Diagram



$\tan\theta = \frac{3}{4}$



$\therefore \cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

Since A is moving, Friction must be maximum. $F_{max} = \mu R$

$F = \mu R$

$= \mu \times 2g \times \frac{4}{5} = \frac{8}{5}\mu g$ friction. B1

the total KE gained as both go from rest ($0\,\text{ms}^{-1}$) to $4.5\,\text{ms}^{-1}$:

$\Delta KE = \frac{1}{2}(6)(4.5)^2 = 60.75\,\text{J}$

$2+4$ kg B1

from (a), GPE lost = 82.3J

$\therefore$ in our equation: GPE lost − WD by friction = KE gained M1

$82.3 - \frac{8}{5}\mu g = 60.75$ substitute

$82.3 - 60.75 = 47.04\mu$ A1 B1

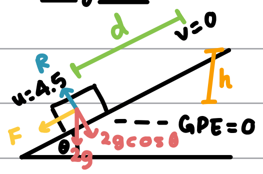
$\mu = 0.459$ to 3sf A1

P 7 2 0 9 2 R A 0 2 2 3 2

Question 6 continued

(c) ★Look at WE-formulae and explanation above in part (b).

Diagram



$\sin\theta = \frac{3}{5} = \frac{h}{d}$

$h = \frac{3}{5}d$

h in terms of d

Substitute into W-E formula M1

$KE_I + \cancel{GPE_I} - W_F = KE_F + GPE_F$

$\frac{1}{2}(2)(4.5)^2 - \underbrace{d(\mu \times 2g\cos\theta)}_{\text{work done by friction.}} = \frac{1}{2}(2)(0) + 4g(\frac{3}{5}d)$ A1

and $F_{max} = \mu R$ due to movement.

$\frac{81}{4} = 16.9 \times \frac{3}{5} \times d + 19.6 \times \frac{4}{5} \times \mu d$ we found μ in (b) A1

plug this into your calculator

$d = 1.07\text{m}$ to 3sf A1

(Total for Question 6 is 13 marks)

7. A spring of natural length a has one end attached to a fixed point A. The other end of the spring is attached to a package P of mass m.
The package P is held at rest at the point B, which is vertically below A such that $AB = 3a$.
After being released from rest at B, the package P first comes to instantaneous rest at A.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$

(5)

(b) (i) Show that P attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of a and g.

(6)

In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b).

(1)

Question 7 continued

(a)

★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction).

★ Remember the work-energy formulae:

Either: $$\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} + \underbrace{EPE_i}_{\text{initial elastic potential}} = \underbrace{KE_F}_{\text{final kinetic}} + \overbrace{GPE_F}^{\text{final grav. potential}} + \underbrace{EPE_F}_{\text{final elastic potential}} + \underbrace{\text{WD against friction}}_{\text{work lost to friction}}$$

OR: $$\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} + \underbrace{EPE_i}_{\text{initial elastic potential}} - \underbrace{\text{WD by friction}}_{\text{we subtract this since it leaves the system as heat!}} = \underbrace{KE_F}_{\text{final kinetic}} + \overbrace{GPE_F}^{\text{final grav. potential}} + \underbrace{EPE_F}_{\text{final elastic potential}}$$

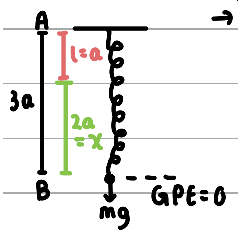
★ Formulae for KE, GPE and EPE:

$KE = \frac{1}{2}mv^2$ — m: mass, v: velocity

$GPE = mgh$ — m: mass, h: change in height, $g = 9.8\,ms^{-2}$

$EPE = \frac{\lambda x^2}{2l}$ — λ: modulus of elasticity, x: extension of string/spring, l: natural length of the string/spring

Diagram



→ it moves from rest to rest ∴ $\Delta KE = 0$

Substitute:

$$\underbrace{\frac{\lambda(2a)^2}{2a} + mg\cancel{(0)}^{0}}_{\text{at B}} = \underbrace{\frac{\lambda(a)^2}{2a} + mg(3a)}_{\text{at A}}$$ Solve for λ M1A1

$$\frac{\cancel{4}^{2}a^2\lambda}{\cancel{2a}} = \frac{\lambda a^2}{2a} + 3mga$$ M1A1

$$2a\lambda = \frac{\lambda a}{2} + 3mga$$

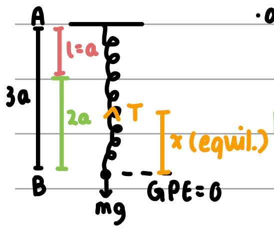
$$\frac{3}{2}a\lambda = 3mga$$ cancel a's

$$\lambda = \frac{2}{\cancel{3}} \times \cancel{3}mg$$

$$\boxed{\lambda = 2mg}$$ hence shown A1

Question 7 continued

(b) At equilibrium the speed is maximum and $\Sigma F_y = 0$.



as $\Sigma F_y = 0$:

$T = mg$

Formula for T:

$T = \frac{\lambda x}{l}$ extension when speed is maximum

$T = \frac{2mgx}{a}$ M1

$\cancel{mg} = \frac{2\cancel{mg}x}{a}$ cancel mg

$a = 2x$

$x = \frac{a}{2}$ extension! A1

Now we can use the WE principle to get the maximum speed (occurs at equilibrium).

★look at WE-formulae and explanation above in part (a).

Substitute:

we want this!

$$\underbrace{\frac{2mgx^2}{2a} + mg\underbrace{(2a-x)}_{\text{distance moved from B to equilibr.}} + \frac{1}{2}mv^2}_{\text{at equilibrium}} = \underbrace{\frac{2mg(2a)^2}{2a}}_{\text{at B}} \quad \text{M1A1}$$

Now solve for V:

$x = \frac{a}{2}$

$$\frac{mgx^2}{a} + 2mga - mgx + \frac{1}{2}mv^2 = \frac{4a^2mg}{a}$$

$$\frac{\cancel{m}g\left(\frac{a}{2}\right)^2}{a} + 2\cancel{m}ga + \frac{1}{2}\cancel{m}v^2 = 4a\cancel{m}g + \cancel{m}g\left(\frac{a}{2}\right) \quad \text{cancel m's}$$

$$\frac{a^2g}{4a} + 2ag + \frac{1}{2}v^2 = 4ag + \frac{1}{2}ag$$

$$\frac{1}{4}ag + 2ag + \frac{1}{2}v^2 = \frac{9}{2}ag \quad \text{A1}$$

$$\frac{1}{2}v^2 = \frac{9}{2}ag - \frac{9}{4}ag$$

$$\frac{1}{2}v^2 = \frac{18}{4}ag - \frac{9}{4}ag$$

$$\frac{1}{2}v^2 = \frac{9}{4}ag$$

$$v^2 = \frac{9}{2}ag$$

$$v = 3\sqrt{\frac{ag}{2}}\ \text{ms}^{-1} \quad \text{max speed} \quad \text{A1}$$

(c) Would need to:

→ consider GPE of spring in the energy equation

→ consider KE of spring in the energy equation

→ the extension at equilibrium will be different B1

Question 7 continued

(Total for Question 7 is 12 marks)

P 7 2 0 9 2 R A 0 2 7 3 2

8.

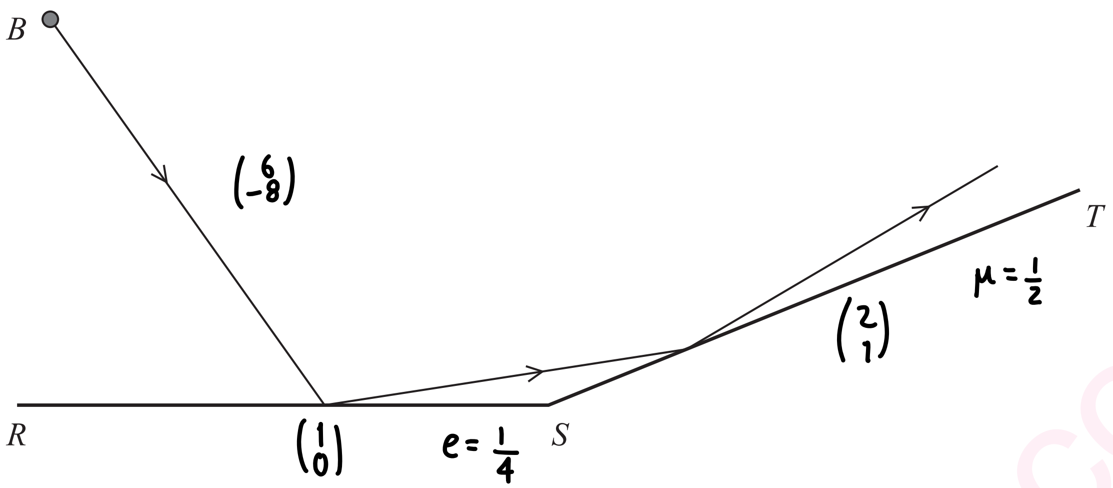


Figure 5

Figure 5 represents the plan view of part of a smooth horizontal floor, where RS and ST are smooth fixed vertical walls. The vector $\overrightarrow{RS}$ is in the direction of **i** and the vector $\overrightarrow{ST}$ is in the direction of $(2\mathbf{i} + \mathbf{j})$.

A small ball B is projected across the floor towards RS. Immediately before the impact with RS, the velocity of B is $(6\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$. The ball bounces off RS and then hits ST.

The ball is modelled as a particle.

Given that the coefficient of restitution between B and RS is e,

(a) find the full range of possible values of e.

(3)

It is now given that $e = \frac{1}{4}$ and that the coefficient of restitution between B and ST is $\frac{1}{2}$

(b) Find, in terms of **i** and **j**, the velocity of B immediately after its impact with ST.

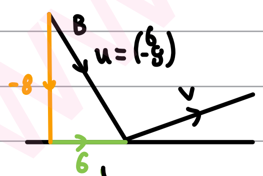
(7)

(a) For the ball to collide with ST, the vector v must have a smaller j:i ratio with its components than the wall ST.

For the wall ST the ratio is: $\frac{j}{i} = \frac{1}{2}$

For B:

Diagram



Perpendicular to the wall use NLR and change direction:

$-8 \times e \times -1 = 8e$ j-component B1

$\therefore$ B's velocity after colliding with RS:

$v = \begin{pmatrix}6\\8e\end{pmatrix}$

parallel to the wall there is no change $\therefore$ i component B1

The ratio is: $\frac{j}{i} = \frac{8e}{6}$

Compare the ratios:

$\frac{8e}{6} < \frac{1}{2} \rightarrow e < \frac{3}{8}$

As e is a coefficient of restitution it needs to be larger than 0.

$0 < e < \frac{3}{8}$ Range for e B1

P 7 2 0 9 2 R A 0 2 8 3 2

Question 8 continued

(b) Formulae for vector collisions:

$\underset{\sim}{u} \cdot \underset{\sim}{w} = \underset{\sim}{v} \cdot \underset{\sim}{w}$ → parallel vector to wall

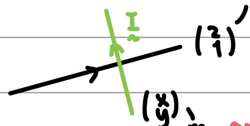
initial speed ($\underset{\sim}{u}$), final speed ($\underset{\sim}{v}$)

$-e\,\underset{\sim}{u} \cdot \underset{\sim}{I} = \underset{\sim}{v} \cdot \underset{\sim}{I}$ → perpendicular vector to wall

I. Wall vectors

$\underset{\sim}{w} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

to get $\underset{\sim}{I}$:



$m_1 = \frac{1}{2}$ gradient

the two are perpendicular, ∴ use $m_1 \times m_2 = -1$ to get m_2.

$\frac{1}{2} \times m_2 = -1$

$m_2 = -2$

$\frac{y}{x} = -2 \rightarrow y = -2x \rightarrow$ vector: $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ perpendicular to the wall M1

$\therefore \underset{\sim}{I} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

II. Velocity vectors

$\underset{\sim}{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ we're solving for this

use (a) to get $\underset{\sim}{u}$.

$\underset{\sim}{u} = \begin{pmatrix} 6 \\ 8e \end{pmatrix}$ given that $e = \frac{1}{4}$

$\therefore \underset{\sim}{u} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

Put all our parameters in one place:

$\underset{\sim}{w} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad \underset{\sim}{u} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$\underset{\sim}{I} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\quad \underset{\sim}{v} = \begin{pmatrix} a \\ b \end{pmatrix}$

$e = \mu = \frac{1}{2}$

Substitute into formulae: scalar product: $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = ac + bd$

→ Parallel to wall: $\underset{\sim}{u} \cdot \underset{\sim}{w} = \underset{\sim}{v} \cdot \underset{\sim}{w}$

$\begin{pmatrix} 6 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ M1

$12 + 2 = 2a + b$

$14 = 2a + b$ Eq.1 A1

→ Perpendicular to wall: $-e\,\underset{\sim}{u} \cdot \underset{\sim}{I} = \underset{\sim}{v} \cdot \underset{\sim}{I}$

$-\frac{1}{2} \times \begin{pmatrix} 6 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ M1

$-\frac{1}{2}(6 - 4) = a - 2b$

$-1 = a - 2b$ Eq.2

A1

Question 8 continued

Solve simultaneously Eq1 and Eq2: M1

sub. into Eq1 $a = 2b - 1$ from Eq2

$14 = 2(2b-1) + b$

$14 = 5b - 2$

$16 = 5b$

$b = \frac{16}{5}$ b

substitute back:

$a = 2 \times \frac{16}{5} - 1$

$= \frac{32}{5} - 1$

$a = \frac{27}{5}$ a

∴ the velocity of B after:

$\underset{\sim}{v} = \begin{pmatrix} \frac{27}{5} \\ \frac{16}{5} \end{pmatrix} \text{ms}^{-1}$

$\underset{\sim}{v} = \left(\frac{27}{5}i + \frac{16}{5}j\right) \text{ms}^{-1}$ A1

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

P 7 2 0 9 2 R A 0 3 2 3 2